REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

1[65–01, 65N30].—R. E. WHITE, *An Introduction to the Finite Element Method with Application to Nonlinear Problems*, Wiley-Interscience, New York, 1985, x + 354 pp., $23\frac{1}{2}$ cm. Price $39.50.

This text is intended for a semester course for graduate level engineers. It has three clear objectives, as stated in the Preface: (1) Giving the student the ability to modify existing finite element codes or to create new codes. (2) Giving the student some appreciation for the error estimates, and (3) Giving a summary and illustration of nonlinear algorithms. The author pursues these objectives without getting sidetracked, and he succeeds in reaching his first and third goal. (More about the second objective below.)

Apart from the introductory first chapter and a chapter on error estimates, the emphasis is on problems in two space dimensions. Applications, occupying Chapters 8 and 9, are beautifully selected. The reader of this review may judge for himself: Nonlinear heat conduction, Burgers' equation, incompressible viscous fluid flow, Stefan problem, obstacle problem for a string (for motivation), elliptic variational inequalities, fluid flow in a porous medium, and parabolic variational inequalities. Four appendices give computer programs for steady-state heat conduction, heat flow in a resistance transducer, solidification of water in a channel, and steady state flow in a porous medium.

Indeed, an interesting list!

The second objective stated above is to give the student some appreciation for error estimates. I proceed to elaborate on why I think the author has failed to do so. (I note in passing that in the exemplarily brief Introduction to this work, the second objective has disappeared. Only the first and third objectives from the Preface survive in explicitly or implicitly stated form.) To set the stage, I first describe some results from Chapter 5 concerned with error estimates for piecewise linear splines in a two-point boundary value problem. The main result, stated already in the first paragraph of the chapter and then in Theorem 5.6.1, says that the pointwise displacement error is of first order in the mesh-spacing parameter. On p. 138 a second-order displacement error estimate in L_2 is given, with the following remark: "Note that the norm $\|\cdot\|_{L_2}$ is weaker than absolute value, but the order is now 2 and not 1."

In my experience (I have taught similar courses), engineering students appreciate error estimates when they realize how valuable they are in practice, namely for testing that codes (including canned programs) are correct. Merely testing *convergence* on a model problem is not enough. *Rates of convergence* must be tested on smooth model problems, at least. If theory predicts fourth-order convergence and your program gives only first-order convergence, then there is an error in your program. (This is an example of what happened to one of my students; another student observed third-order rather than fourth. Both errors were rather subtle but could be found after further suitable experimentation. These students will, I hope, never use a program without testing it against theory on simple models.)

To repeatedly state, as this text does on three occasions, a first-order pointwise result for displacements is misleading. A student who does convergence rate tests on smooth problems and finds (correctly) second-order for piecewise linear approximations is likely to be confused.

(A second-order result for pointwise displacement error is stated, without proof, as Corollary 1 on p. 133. Apparently, in the five years that the author has taught this course prior to publishing the present text, no student got fascinated enough with error estimates to ask how this corollary would follow from Theorem 5.6.1. Well, it doesn't; completely different techniques are required to prove it.)

Another point where the theory as presented may mislead a beginner occurs in Chapter 6, devoted to time-dependent problems. Stability is discussed, mainly in the L_∞-norm, and then the Lax Equivalence Theorem is stated in its usual formulation, with the sole comment that it is an "important theorem". The student is likely to be confused when (if ever) he learns that, e.g., the widely used Lax-Wendroff scheme is unstable (in L_∞). At the introductory level, Lax's theorem is one of the most misunderstood results ever, and the present text continues the tradition.

In summary, why I think that the author has failed to impart any understanding of the role of error estimates: They are not put to any use, practical or otherwise. Engineering students are likely to be moderately enthusiastic about this approach.

It is traditional for a reviewer to present his minor differences of opinion with the author:

In the first and second sections of the first chapter, the author starts with a two-point boundary value problem and then considers an energy functional to be minimized. The same order of business occurs in the second section of the second chapter for a plane Poisson problem. Engineering students are likely to appreciate the reverse order: Minimizing the energy functional is the fundamental physical principle and the two-point boundary value problem or Poisson problem follows as a consequence. One may then point out that the finite element method is one step closer to the fundamental principle than, say, some finite difference method!

The remark on p. 251 on the Courant number condition (I prefer the term Courant-Friedrichs-Lewy number to honor all involved) ought to be amplified. Students immediately understand domain of dependence considerations.

In Section 8.4, (8.4.5), the viscous Navier-Stokes equations are presented with the wrong boundary conditions (inviscid). This is rectified six pages later (p. 261) with an explanation that I defy anyone to make sense of!

The method of lumping, described in Remark 1 on p. 265, should be given a reference, if not a brief explanation of why it works.

The relation between the approximate velocity space and the approximate pressure space alluded to on p. 267 also merits a reference. The explanation offered, namely "This is necessary because in (8.5.4)–(8.5.6) the approximation of the first derivatives of u and v and the approximation of P should have the same order", is nonsense. The situation is reasonably well understood by Numerical Analysts but often a mystery to engineering students.

The students may question the explanation on p. 276 of why test functions in the weak formulation of the enthalphy formulation of the Stefan problem suddenly need to depend on both space and time. After all, these test functions were time-independent in previous nonlinear parabolic problems!

Concluding this review, I congratulate Professor White on a fine text, written with clear perspectives which he sticks to throughout. Five years of classroom experience shows! The emphasis on interesting nonlinear problems in particular sets this book apart from the crowd of introductory texts on finite elements. Also, it is a handsome volume with typography that pleases the eye. I recommend it for its intended purpose without hesitation. If, in a next edition, the author elucidates the practical importance of knowing the correct rate of convergence, it may become the best introductory book on finite elements on the market for an engineering or physics audience.

And, for a second ending of my review: I had fun reading this book!

L. B. W.

2[65–01, 65N30].—R. WAIT & A. R. MITCHELL, *Finite Element Analysis and Applications*, Wiley, Chichester, 1985, xii + 260 pp., $23\frac{1}{2}$ cm. Price $35.00.

This is a comprehensive text on finite elements. It is intended for "final year undergraduate or first year postgraduate students in mathematical sciences or engineering". Further, "no specialized mathematical knowledge beyond a familiarity with calculus and elementary differential equations is assumed".

I wish to add to that a general requirement of mathematical maturity: The Introduction breezes through function spaces, Hilbert spaces, linear operators, Riesz' representation theorem, Lax-Milgram's lemma, monotone operators, Sobolev spaces, trace theorems and other standard material in twelve pages. The second chapter covers extrema of integrals, Euler-Lagrange equations, constrained extrema, possibly with boundary conditions, Hamilton's principle, and dual variational principles in fourteen pages.

It is clear that the students referred to above are great students in Britain, not our typical students in a US university.

As already noted, the text is comprehensive, indeed almost encyclopedic. This leads to a lack of clear objectives (other than to "understand finite elements") that I suspect US undergraduate students or engineering graduate students will not be enthusiastic about.

The book succeeds in being comprehensive in 251 pages. It should serve well as a text for a graduate course in Mathematics or for self study for a mathematically mature person really interested in learning the subject. The style is readable.